Spezielle Taylor-Reihen

$$(1+x)^s = \sum_{k=0}^{\infty} \binom{s}{k} x^k = 1 + sx + \frac{s(s-1)}{2}x^2 + \cdots \qquad |x| < 1$$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2} + \cdots \qquad x \in \mathbb{R}$$

$$\ln(1+x) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^k}{k} = x - \frac{x^2}{2} + \frac{x^3}{3} \pm \cdots \qquad -1 < x \leq 1$$

$$\sin x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!} = x - \frac{x^3}{6} + \frac{x^5}{120} \pm \cdots \qquad x \in \mathbb{R}$$

$$\cos x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!} = 1 - \frac{x^2}{2} + \frac{x^4}{24} \pm \cdots \qquad x \in \mathbb{R}$$

$$\tan x = \sum_{k=1}^{\infty} (-1)^{k-1} \frac{4^{2k} - 4^k}{(2k)!} B_{2k} x^{2k-1} = x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots \qquad |x| < \frac{\pi}{2}$$

$$\arcsin x = \sum_{k=0}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2k-1)}{2 \cdot 4 \cdot 6 \cdots (2k)} \frac{x^{2k+1}}{2k+1} = x + \frac{x^3}{6} + \frac{3x^5}{40} + \cdots \qquad |x| < 1$$

$$\begin{aligned} \arccos x &= \frac{\pi}{2} - \sum_{k=0}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2k-1)}{2 \cdot 4 \cdot 6 \cdots (2k)} \frac{x^{2k+1}}{2k+1} \\ &= \frac{\pi}{2} - \left(x + \frac{x^3}{6} + \frac{3x^5}{40} + \cdots \right) \qquad |x| < 1 \end{aligned}$$

$$\arctan x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1} = x - \frac{x^3}{3} + \frac{x^5}{5} \pm \cdots \qquad |x| < 1$$